INFNCA-TH9909
October 1999

Reply Comment on "Entropy of 2D black holes from counting microstates"

Mariano Cadoni[a,c,*] and Salvatore Mignemi[b,c,**]

[a] *Dipartimento di Fisica, Università di Cagliari,*

Cittadella Universitaria, 09042, Monserrato, Italy.

[b] *Dipartimento di Matematica, Universitá di Cagliari,*

viale Merello 92, 09123, Cagliari, Italy.

[c] *INFN, Sezione di Cagliari.*

Abstract

We show that the arguments proposed by Park and Yee against our recent derivation of the statistical entropy of 2D black holes do not apply to the case under consideration

PACS: 04.70. Dy, 04.50. +h

---

*E-Mail: CADONI@CA.INFN.IT

**E-Mail: MIGNEMI@CA.INFN.IT

In a recent Comment [1] Park and Yee claimed that derivation of the statistical entropy of 2D (two-dimensional) black holes proposed by us in Ref. [2] is plagued by an error. In this Reply Comment we show that the arguments used by Park and Yee in Ref. [1] against our derivation do not apply to our case. Before going into the details of the confutation of the claim of Ref. [1], let us briefly explain the arguments of Park and Yee.

In our attempt to calculate the statistical entropy of the 2D anti-de Sitter (AdS) black hole along the lines of Ref. [3] we found a major difficulty: owing to the dimension of the boundary, the charges $J[\chi]$ (Eq. (18) of Ref. [2]) do not support a realization of the Virasoro algebra (the asymptotic symmetries of 2D AdS space). This problem is not a peculiarity of the 2D case but shows up also in higher dimensions [4]. To solve the problem we proposed to define the new, time-integrated, generators $\hat{J}[\chi]$ (Eq. (22) of Ref. [2]). Moreover, we were able to show that the Dirac bracket algebra of the charges $\hat{J}[\chi]$ gives a central extension of the Virasoro algebra and to calculate its central charge. To compute the central charge of the algebra we used the equation:

$$\widehat{\delta_\omega J[\chi]} = \hat{J}[[\chi, \omega]] + c(\chi, \omega), \tag{1}$$

where the hat has the meaning of an overall time-integration. Park and Yee claim that the left-hand side of Eq. (1) can not be written as

$$\{\hat{J}[\chi], \hat{J}[\omega]\}_{DB}, \tag{2}$$

thus invalidating our result that the charges $\hat{J}$ span a representation of the Virasoro algebra.

The demonstration of Park and Yee relies on the two assumptions that need to be generalized if one wants to interpret consistently the time-integrated charges $\hat{J}$ as generators of a algebra. First, the generators of the asymptotic symmetry can no longer be identified with the phase space functionals $H[\chi]$ (Eq. (17) of Ref. [2]), but rather with the time-integrated ones $\hat{H}[\chi]$. Second, the usual definition of the Poisson brackets, as brackets evaluated at equal times, has to be generalized in order to allow for general brackets

$$\{\hat{H}[\chi], \hat{H}[\omega]\}_{PB}, \tag{3}$$

where $\hat{H}[\chi]$ is a time-integrated functional. This generalization of objects of the canonical formalism is implicitly contained in the definition of the charges $\hat{J}[\chi]$ [5]. Moreover this is what is needed in order to recognize Eq. (1) as a canonical realization of the asymptotic symmetries. We do not know if in this framework the charges $\hat{J}[\chi]$ have a sensible interpretation as Noether charges. This is irrelevant for our purposes since we are just looking for a canonical realization of the asymptotic symmetries that allows us to perform a computation of the central charge of the algebra.

Using the previously defined generalized notions of canonical generators and Poisson brackets we can easily prove that the left-hand side of Eq. (1) can be written as a Dirac bracket algebra. One just needs to compute explicitly the brackets $\{\hat{H}[\chi], \hat{H}[\omega]\}_{PB}$. One finds [5]:

$$\{\hat{H}[\chi], \hat{H}[\omega]\} = \hat{H}[[\chi, \omega]] + c(\chi, \omega) \tag{4}$$

where the central charge has exactly the same value found in Ref. [2]. Fixing the gauge so that the constraints hold strongly and using Eq. (17) of Ref. [2], the previous equation implies

$$\{\hat{J}[\chi], \hat{J}[\omega]\}_{DB} = \hat{J}[[\chi, \omega]] + c(\chi, \omega). \tag{5}$$

Comparing this equation with Eq. (1), it follows immediately that the left-hand side of the latter can be written as the Dirac bracket in Eq. (2).

Let us now show explicitly that the calculations used in Ref. [1] by Park and Yee to support their claim are inconsistent with our definitions of generators and Poisson brackets. From equation (4) it follows immediately that the canonical generators of the Virasoro algebra are the functionals $\hat{H}[\chi]$ rather then $H[\chi]$. Therefore Eq. (6) of Ref. [1], which is the starting point of the demonstration of Park and Yee, does not apply. The right equation to be used here is instead:

$$\{J[\chi], \hat{H}[\omega]\}_{DB} = \{J[\chi], \hat{J}[\omega]\}_{DB} = \delta_\omega J[\chi]. \tag{6}$$

Following Yee and Park we perform now the time integration of Eq. (6). The left-hand side becomes

$$\frac{\lambda}{2\pi} \int_0^{\frac{2\pi}{\lambda}} dt' \{J[\chi(t')], \hat{J}[\omega]\}_{DB} = \{\hat{J}[\chi], \hat{J}[\omega]\}_{DB}, \tag{7}$$

from which it follows immediately that the left-hand side of Eq. (1) can be written as a Dirac bracket.

References

[1] Mu-In Park and Jae Hyung Yee, Comments on “Entropy of 2D black holes from counting microstates”, hep-th/9910213.

[2] M. Cadoni and S. Mignemi, Phys. Rev. **D59**, 081501(1999).

[3] A. Strominger, J. High Energy Phys. **02** (1998) 009; J. D. Brown and M. Henneaux, Comm. Math. Phys. **104**, 207 (1986).

[4] S. Carlip, Class. Quant. Grav.**16** (1999) 3327.

[5] M. Cadoni, S. Mignemi, Nucl. Phys. **B557** (1999) 165.